Control of a High Precision Macro-Micro Robotic Manipulator System

Whang Cho*

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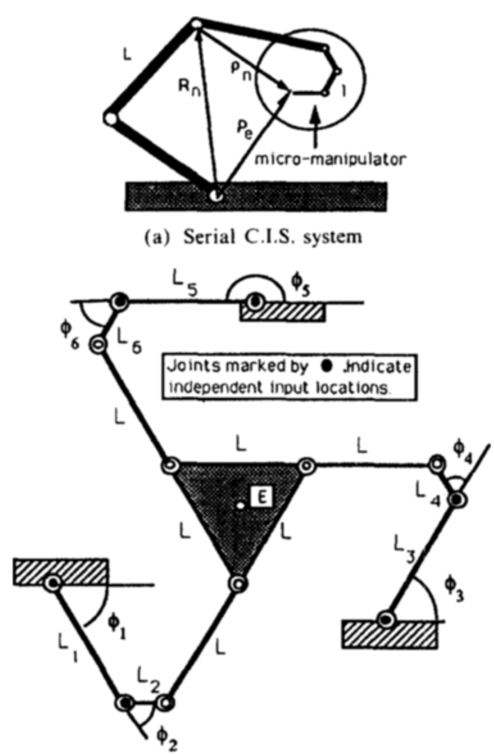
A controller for macro-micro robotic manipulator system in which kinematically independent two robotic sub-systems work together to improve the accuracy of the motion is proposed. A nonlinear feedback linearization scheme is employed as basic architecture for the controller and additional formulations about the controller structure are made to assure the robustness of the overall control action and to restrict the motion of micro sub-system close to its nominal position without causing saturation of joints associated with micro-robot.

Key Words : MAMI Robotic System, Feedback Linearization

1. Introduction

It is well known that it is very difficult to increase the operational accuracy of conventional robotic manipulators which are designed for large workspace operations. This is mainly because of the various limits involved in mechanical hardware, e.g., vibration, backlashes, etc. and consequently any attempts to design a proper controller which can overcome these difficulties usually fail. To increase the accuracy of a robotic manipulator while maintaining large workspace, robotic manipulators with the idea of control in the small (C.I.S.) in their structure is very promising (Egelend, 1987). Figure 1. shows conceptual views of two types of hybrid robotic manipulator systems. As shown in the Fig. 1, a hybrid robotic manipulator system has two sub-robotic systems, one for the generation of large but coarse motions and the other for the high-precision small range motions. The former robotic subsystem may be called macro-robot and the latter micro-robot system. To be able to rapidly correct the motion error of macro robot, the dynamic bandwidth of micro robot system is usually designed to be much larger than that of macro system.



(a) Serial C.I.S. system

(b) Parallel C.I.S. system

Fig. 1 Conceptual view of macro-micro robotic manipulator system.

Since a macro-micro (MAMI) robotic system contains two different sub-dynamic systems which

* Dept. of Control & Instrumentation, Institute of New Technology Kwangwoon University, Seoul, Korea

are kinematically independent of each other, various control schemes can be proposed utilizing the kinematic redundancy of the system. Many approaches have been reported in the literature along this line for general kinematically redundant robotic manipulator systems (Klein and Huang 1983; Yoshikawa, 1985; Hollerbach and Suh, 1987; Sallisbury and Abramowitz, 1985). But due to the special characteristics of the MAMI system (i.e., the "smallness" of the kinematic redundancy in position level), the expected improvements of the system performances through these optimization techniques will be minimal. This assertion can be appreciated to be true by considering kinematic and dynamic manipulability (Yoshikawa, 1985) of the MAMI system. Therefore, a control system architecture which directly utilizes the kinematic and dynamic characteristics of the MAMI system is favored over the passive utilization of the structure of the MAMI system through various motion optimizations.

Recalling that the MAMI system is proposed for, and designed to operate in high precision task environments, the action of required controller should be very robust under various uncertainties which may be encountered in the phases of actual operations. If this is not the case, the whole purpose of the MAMI system becomes less meaningful considering the additional complexities introduced into the system. This implies that in designing a controller for the MAMI system, the robustness of the controller should be the primary concern.

There is an additional operational requirement of a controller for MAMI systems. Since the work space of the small sub-dynamic system is very restricted and, therefore, its motion can be easily saturated, extra control actions should be employed to keep the motion of small system close to its nominal configuration.

In this paper the nonlinear feedback linearization (Kreutz, 1986) scheme will be used as basic control architecture. Successive modifications of the control structure will be made to assure the robustness of the overall control action and to restrict the motion of small subsystem close to its nominal position.

Nonlinear feedback control schemes proposed in the literature for making the input-output behavior of a robotic manipulator equivalent to a decoupled linear system can be classified as follows: Computed Torque technique (Markiewicz, 1973) and its extension to task space (Khatib, 1987), resolved acceleration scheme (Luh et al., 1980), and general nonlinear decoupling theory (Meyer, 1980, 1984 and Su, 1982; Su and Hunt, 1985; Hunt et al., 1983). The basic difference between the computed torque technique and the resolved acceleration technique lies in the resolution of motion errors. The computed torque technique evaluates motion errors directly in joint space while the resolved acceleration scheme detects the errors in task space allowing tighter control of the actual end-effector motions.

The application of nonlinear decoupling theory to general nonlinear systems is rather involved due to required differential geometry arguments. But it can be shown (Dwyer, 1984; Ha and Gilbert, 1987; Gilbert and Ha, 1984) that there exists a natural class of state and input transformations applicable to robotic manipulator systems, which can be seen as a generalization of the resolved acceleration scheme.

Numerous robust control schemes have been reported in the literature for the control of robotic manipulators (Arimoto and Miyazaki, 1984 and 1986; Corless and Leitman, 1984; Gilbert and Ha, 1984; Ha and Gilbert, 1987; Mills and Goldenberg, 1986; Spong et al, 1984; Lim and Eslami, 1985 and 1987). They in general utilize large feedback gains to make the system robust under various disturbances. In this paper, the primary issue is how to design a robust controller which is easily implemented and does not require excessive amounts of control efforts. This implies that bounds on various uncertainties must be estimated as tightly as possible. The following formulation can be viewed as an extension of the work pioneered by Leitman (1981), which are successively developed by Corless and Leitman (1984), Chouinard at al. (1985), Ha and Gilbert (1987), Marnish and Leitman (1982).

2. Architecture of Proposed Controller

The resolved acceleration scheme (Luh et al., 1980) can be put into a more general format as follows: Assume the dynamics of the system is expressed in joint space as

$$[I^*_{\phi\phi}]\ddot{\phi} + \dot{\phi}^T \otimes \{P^*_{\phi\phi\phi}\} \otimes \dot{\phi} = T_\phi \tag{1}$$

where $\phi \in R^n$ denotes the generalized coordinate vector, $[I^*_{\phi\phi}] \in R^{n\times n}$ the inertia matrix, $\{P^*_{\phi\phi\phi}\} \in R^{n\times n\times n}$ three dimensional inertia related tensor, T_ϕ generalized torque, $\otimes$ generalized matrix multiplication operator which performs matrix multiplications for each plane matrix of corresponding three dimensional tensor (see Cho et al., 1989; Cho, 1994) , $n = n_l + n_s$ the total degrees of freedom with n_l and n_s denoting the degrees of freedom of macro subsystem for large motion and micro subsystem for small motion, respectively.

Suppose also that the kinematic relations between the end-effector output motion $u(t) \in R^m$ and the joint input motion $\phi(t) \in R^n$ may be written as

$$\dot{u} = [G^u_\phi]_l\dot{\phi}_l + [G^u_\phi]_s\dot{\phi}_s \tag{2}$$

$$\begin{aligned}\ddot{u} =& [G^u_\phi]_l\ddot{\phi}_l + [G^u_\phi]_s\ddot{\phi}_s \\ &+ \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ll} \otimes \dot{\phi}_l + \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ls} \otimes \dot{\phi}_s \\ &+ \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{sl} \otimes \dot{\phi}_l + \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{ss} \otimes \dot{\phi}_s\end{aligned} \tag{3}$$

where $[G]$'s denote Jacobian matrices and $\{H\}$'s stand for Hessian tensors with proper dimensions. Then, we have the following proposition (Kreutz, 1986).

Proposition 1 *The feedback transformation T_ϕ defined as*

$$T_\phi = [I^*_{\phi\phi}]\xi + \dot{\phi}^T \otimes \{P^*_{\phi\phi\phi}\} \otimes \dot{\phi} \tag{4}$$

with ξ being any solution to

$$\begin{aligned}\Big[[G^u_\phi]_l \otimes [G^u_\phi]_s\Big]\xi =& \eta - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ll} \otimes \dot{\phi}_l - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ls} \otimes \dot{\phi}_s \\ &- \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{sl} \otimes \dot{\phi}_l \\ &- \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{ss} \otimes \dot{\phi}_s\end{aligned} \tag{5}$$

reduces the dynamics given in Eq. (1) to a simple system

$$\ddot{u} = \eta \tag{6}$$

where the vectors $\eta \in R^m$ and $\xi \in R^n$ are commanded accelerations in task space and joint space, respectively.

The proposition implies that if there is kinematic redundancy, i.e., when $[[G^u_\phi]_l[G^u_\phi]_s]$ is not directly invertible, there can be many different feedback transformations T_ϕ, depending on the particular solution ξ, which leads to the same dynamic behavior in task space as in Eq. (6). A meaningful solution of ξ can be obtained by instantaneously minimizing $\|\xi\|_2^2$, where $\|\cdot\|_2$ denotes the Euclidean norm of a vector.

A direct dynamic level optimization rather than optimization at the kinematic level can also be accomplished as follows: Simple rephrasing of proposition 1 yields the following (Cho, 1989).

Proposition 2 *The feedback transformation T_ϕ which is any solution to*

$$\begin{aligned}&\Big[[G^u_\phi]_l[G^u_\phi]_s\Big][I^*_{\phi\phi}]^{-1}T_\phi \\ &= \eta + \Big[[G^u_\phi]_l[G^u_\phi]_s\Big][I^*_{\phi\phi}]^{-1}\Big(\dot{\phi}^T \otimes \{P^*_{\phi\phi\phi}\} \otimes \dot{\phi}\Big) \\ &\quad - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ll} \otimes \dot{\phi}_l - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ls} \otimes \dot{\phi}_s \\ &\quad - \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{sl} \otimes \dot{\phi}_l - \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{ss} \otimes \dot{\phi}_s\end{aligned} \tag{7}$$

reduces the system given in Eq. (1) to one given in Eq. (6).

Equation (7) can also be solved for T_ϕ by instantaneously minimizing $\|T_\phi\|_2^2$.

For a given MAMI system, the proper kinematic or dynamic optimization criteria which would fully utilize its structural characteristics is not obvious. Therefore, a more direct way of resolving the kinematic redundancy of the MAMI system is desired. One scheme is proposed as follows: Decompose the joint and task space commanded acceleration as $\xi^T = (\xi_l^T \xi_s^T)$, $\eta^T = \eta_l^T + \eta_s^T$ with $\xi_l \in R^{n_l}$, $\xi_s \in R^{n_s}$, η_l, $\eta_s \in R^m$. Then, we have

Proposition 3 *A particular solution ξ to Eq. (5) obtained as*

$$\xi_l = [G^u_\phi]_l^{-1}\Big(\eta_l - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ll} \otimes \dot{\phi}_l\Big) \tag{8}$$

$$\begin{aligned}\xi_s = [G^u_\phi]_s^{-1}\Big(&\eta_s - \dot{\phi}_l^T \otimes \{H^u_{\phi\phi}\}_{ls} \otimes \dot{\phi}_s - \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{sl} \otimes \dot{\phi}_l \\ &- \dot{\phi}_s^T \otimes \{H^u_{\phi\phi}\}_{ss} \otimes \dot{\phi}_s\Big)\end{aligned} \tag{9}$$

reduces the joint space dynamics to task space dynamics

$$\ddot{u}=\eta_l+\eta_s \qquad (10)$$

The proof of this proposition is simple and, therefore, omitted (Cho, 1989). Now, define the task space commanded acceleration η as

$$\eta_l=\ddot{u}_d+(\ddot{u}_d-\ddot{\hat{u}})+[K_v]_l(\dot{u}_d-\dot{\hat{u}}) \\ +[K_p]_l(u_d-\hat{u}) \qquad (11)$$

$$\eta_s=[K_v]_s(\dot{u}_d-\dot{u})+[K_p]_s(u_d-u) \qquad (12)$$

where $u_d(t)\in R^m$ is the desired task space motion planned off-line and $\hat{u}(t)\in R^m$ is the virtual motion of the end-effector defined as

$$\hat{u} = f(\phi_l,\phi_{so}) \qquad (13)$$

$$\dot{\hat{u}} = [G_\phi^u(\phi_l,\phi_{so})]_l\dot{\phi}_l \qquad (14)$$

with ϕ_{so} denoting the nominal configuration of the small system. Note that $\hat{u}(t)$ is calculated using the actual (i.e., current) $\phi_l(t)$ but with $\phi_s(t)$ setting equal to ϕ_{so} and fixed. This step reflects the efforts to prevent frequent saturation of small joints. Finally, feedback gain matrices $[K_p]_l$, $[K_v]_l$, $[K_p]_s$, $[K_v]_s\in R^{m\times m}$ are to be defined such that task space motion is asymptotically stable, assuming no modeling errors exist.

Equations (11), (12) together with Eqs. (8) and (9) imply that the desired motion of the end-effector is generated at the acceleration level by the large joints and the resulting errors are compensated through the action of the small joints. Equation (11) also shows that additional action is made by the large joints so that excursion of the small joints from their nominal positions is kept within an acceptable range.

Substituting Eqs. (11) and (12) into Eq. (10) gives

$$\ddot{e}+[K_v]_s\dot{e}+[K_p]_s\, e=-(\ddot{\hat{e}})+[K_v]_l\dot{\hat{e}} \\ +[K_p]_l\hat{e}) \qquad (15)$$

where the actual (e) and virtual ($\hat{e}$) errors are defined as

$$e=u-u_d \qquad (16)$$

$$\hat{e}=\hat{u}-u_d \qquad (17)$$

Noting that the dynamics on the right hand side of Eq. (15) are independent of those on the left hand side and, therefore, can be made stable by choosing proper gain matrices $[K_p]_l$ and $[K_v]_l$, the overall system becomes asymptotically stable with proper choice of $[K_p]_s$ and $[K_v]_s$. Summarizing the observations made so far, we have

Proposition 4 *The feedback torque T_ϕ which by feedback will linearize the dynamics given in Eq. (1) and stabilize the resulting system in Eq. (6) by proper pole assignment, is given by Eq. (4) with Eqs. (8) to (12).*

A final remark on selecting feedback gain matrices $[K_p]_l$, $[K_v]_l$, $[K_p]_s$, and $[K_v]_s$ seems to be in order. $[K_p]_s$, $[K_v]_s$, should be selected to realize high band-width dynamics on actual error e, while $[K_p]_l$, $[K_v]_l$, are to be chosen to show slow dynamics on the virtual error $\hat{e}$. This is consistent with the operational characteristics of MAMI system, i.e., generation of small high band-width motions in addition to slowly changing large motions.

3. Design of a Robust Controller for MAMI Systems

3.1 Basic formulations

In the previous section the fundamental structure of the resolved acceleration scheme was introduced without employing any rigorous stability and robustness arguments. However, it is intuitively obvious that the scheme will generate stable motion if the modeling errors are very small. Hereafter, a more rigorous formulation toward robust controller design will be developed to be able to accomplish tight tracking of the desired end-effector motion in the presence of appreciable modeling errors.

Suppose the system dynamics are given as in Eq. (1) or, in state space form as

$$\frac{d}{dt}\begin{pmatrix}\phi\\ \dot{\phi}\end{pmatrix}=F(\phi,\ \dot{\phi})+G(\phi)T_\phi \qquad (18)$$

where $F\in R^{2n}$, $G\in R^{2n\times n}$ are defined as

$$F(\phi,\dot{\phi})=\begin{pmatrix}\dot{\phi}\\ -[I_{\phi\phi}^*(\phi)]^{-1}N(\phi,\ \dot{\phi})\end{pmatrix} \qquad (19)$$

$$G(\phi)=\begin{bmatrix} [0] \\ [I^*_{\phi\phi}(\phi)]^{-1} \end{bmatrix} \tag{20}$$

with $[0]\in R^{n\times n}$ denoting the null matrix, and $N(\phi,\dot{\phi})\in R^n$ given by

$$N(\phi,\dot{\phi})=\dot{\phi}^T\otimes\{P^*_{\phi\phi\phi}(\phi)\}\otimes\dot{\phi} \tag{21}$$

Assume the following mathematical model of the system is used for control command generation

$$[\tilde{I}^*_{\phi\phi}(\phi)]\ddot{\phi}+\dot{\phi}^T\otimes\{\tilde{P}^*_{\phi\phi\phi}(\phi)\}\otimes\dot{\phi}=T_\phi \tag{22}$$

or, in state space form as

$$\frac{d}{dt}\begin{pmatrix}\phi\\ \dot{\phi}\end{pmatrix}=\tilde{F}(\phi,\dot{\phi})+\tilde{G}(\phi)T_\phi \tag{23}$$

where $\tilde{F}\in R^{2n}$, $\tilde{G}\in R^{2n\times n}$are

$$\tilde{F}(\phi,\dot{\phi})=\begin{pmatrix}\dot{\phi}\\ -[\tilde{I}^*_{\phi\phi}(\phi)]^{-1}\tilde{N}(\phi,\dot{\phi})\end{pmatrix} \tag{24}$$

$$\tilde{G}(\phi)=\begin{bmatrix}[0]\\ [\tilde{I}^*_{\phi\phi}(\phi)]^{-1}\end{bmatrix} \tag{25}$$

with $\tilde{N}(\phi,\dot{\phi})\in R^n$ being defined as

$$\tilde{N}(\phi,\dot{\phi})=\dot{\phi}^T\otimes\{\tilde{P}^*_{\phi\phi\phi}(\phi)\}\otimes\dot{\phi} \tag{26}$$

where the quantities with~ denotes that they are associated with mathematically estimated models.

Define modeling discrepancies, $\Delta F(\phi,\dot{\phi})\in R^{2n}$, and $\Delta G(\boldsymbol{\phi})\in R^{2n\times n}$, between the actual system and the estimated mathematical model as

$$\Delta F(\phi,\dot{\phi})=F(\phi,\dot{\phi})-\tilde{F}(\phi,\dot{\phi}) \tag{27}$$

$$\Delta G(\phi)=G(\phi)-\tilde{G}(\phi) \tag{28}$$

We now show by following a line of reasoning similar to that used in the previous section that there exist state and input transformations which render the system given in Eq. (23) as a simple linear dynamic system in error $\varepsilon\in R^m$ as

$$\frac{d}{dt}\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix}=[A]\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix}+[B]v \tag{29}$$

where $v\in R^m$ is the command input to the system which will be properly defined successively and $[A]\in R^{2m\times 2m}$, $[B]\in R^{2m\times m}$ are defined as

$$[A]=\begin{bmatrix}[0] & [I]\\ [0] & [0]\end{bmatrix} \tag{30}$$

$$[B]=\begin{bmatrix}[0]\\ [I]\end{bmatrix} \tag{31}$$

with null matrix $[0]\in R^{m\times m}$, identity matrix $[I]\in R^{m\times m}$. The error state in Eq. (29) is defined as

$$\varepsilon=\left(e^T\hat{e}^T\right)^T \tag{32}$$

$$=\begin{pmatrix}f(\phi)-u_d\\ f(\phi_l,\phi_{so})-u_d\end{pmatrix} \tag{33}$$

Notice that the error state is augmented to incorporate the desire of restricting the excursion of the small joints to small ranges from their nominal positions. Successive differentiation of ϵ with respect to (w.r.t.) time gives

$$\dot{\varepsilon}=\begin{pmatrix}[G^u_\phi]\dot{\phi}-\dot{u}_d\\ [\hat{G}^u_\phi]\dot{\phi}-\dot{u}_d\end{pmatrix} \tag{34}$$

$$=[\tilde{G}^u_\phi]\dot{\phi}-\begin{pmatrix}\dot{u}_d\\ \dot{u}_d\end{pmatrix} \tag{35}$$

$$\ddot{\varepsilon}=\begin{pmatrix}[G^u_\phi]\ddot{\phi}+\dot{\phi}^T\otimes\{H^u_{\phi\phi}\}\otimes\dot{\phi}-\ddot{u}_d\\ [\hat{G}^u_\phi]\ddot{\phi}+\dot{\phi}^T\otimes\{\hat{H}^u_{\phi\phi}\}\otimes\dot{\phi}-\ddot{u}_d\end{pmatrix} \tag{36}$$

$$=[\tilde{G}^u_\phi]\ddot{\phi}+\dot{\phi}^T\otimes\{\tilde{H}^u_{\phi\phi}\}\otimes\dot{\phi}-\begin{pmatrix}\ddot{u}_d\\ \ddot{u}_d\end{pmatrix} \tag{37}$$

where

$$[\tilde{G}^u_\phi]=\begin{bmatrix}[G^u_\phi]\\ [\hat{G}^u_\phi]\end{bmatrix} \tag{38}$$

$$\{\tilde{H}^u_{\phi\phi}\}=\left\{\{H^u_{\phi\phi}\}\{\hat{H}^u_{\phi\phi}\}\right\} \tag{39}$$

and

$$[\hat{G}^u_\phi]=\left[\left[G^u_\phi(\phi_l,\phi_{so})\right]_l[0]\right] \tag{40}$$

$$[\hat{H}^u_{\phi\phi}]=\begin{Bmatrix}\{H^u_{\phi\phi}(\phi_l,\phi_{so})\}_{ll} & \{0\}\\ \{0\} & \{0\}\end{Bmatrix} \tag{41}$$

with [0]'s and [0]'s denoting null matrices and null tensors of proper dimensions, respectively.

Equations (33) and (35) define the state transformation as

$$\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix}=T(\phi,\dot{\phi},u_d,\dot{u}_d) \tag{42}$$

$$=\begin{pmatrix}f(\phi)-u_d\\ f(\phi_l,\phi_{so})-u_d\\ [\tilde{G}^u_\phi]\dot{\phi}-\begin{pmatrix}\dot{u}_d\\ \dot{u}_d\end{pmatrix}\end{pmatrix} \tag{43}$$

Equations (22), (29), and (37) imply

$$\ddot{\varepsilon}=v \tag{44}$$

$$=[\tilde{G}^u_\phi]\Big([\tilde{I}^*_{\phi\phi}(\phi)]^{-1}T_\phi$$

$$-[\tilde{I}^*_{\phi\phi}(\phi)]^{-1}\tilde{N}(\phi,\dot{\phi})\Big) \quad (45)$$

$$+\dot{\phi}^T\otimes\{\tilde{H}^u_{\phi\phi}\}\otimes\dot{\phi}-\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix} \quad (46)$$

The input transformation between v and T_ϕ implies that

$$v=\Upsilon(\phi,\dot{\phi},\ddot{u}_d)+\Psi(\phi)T_\phi \quad (47)$$

where $\Upsilon\in R^m, \Psi\in R^{m\times n}$ are

$$\Psi(\phi)=[\tilde{G}^u_\phi][\tilde{I}^*_{\phi\phi}(\phi)]^{-1} \quad (48)$$

$$\Upsilon(\phi,\dot{\phi},\ddot{u}_d)=\dot{\phi}^T\otimes\{\tilde{H}^u_{\phi\phi}\}\otimes\dot{\phi}-\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix}$$

$$-[\tilde{G}^u_\phi][\tilde{I}^*_{\phi\phi}(\phi)]^{-1}\tilde{N}(\phi,\dot{\phi}) \quad (49)$$

State and input transformations given in Eqs. (43) and (47), respectively, imply

$$\frac{d}{dt}\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix}=\frac{d}{dt}T(\phi,\dot{\phi},u_d,\dot{u}_d) \quad (50)$$

$$=\left[\frac{\partial T}{\partial\phi}\frac{\partial T}{\partial\dot{\phi}}\right]\begin{pmatrix}\dot{\phi}\\ \ddot{\phi}\end{pmatrix}$$

$$+\frac{\partial T}{\partial u_d}\frac{\partial T}{\partial\dot{u}_d}\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix} \quad (51)$$

$$=\partial(\phi,\dot{\phi})(\tilde{F}+\tilde{G}T_\phi)$$

$$+\left[\frac{\partial T}{\partial u_d}\frac{\partial T}{\partial\dot{u}_d}\right]\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix} \quad (52)$$

$$=[A]T+[B](\Upsilon+\Psi T_\phi) \quad (53)$$

where $\partial(\phi,\dot{\phi})\in R^{2m\times 2n}$ is defined as

$$\partial(\phi,\dot{\phi})=\left[\frac{\partial T}{\partial\phi}\frac{\partial T}{\partial\dot{\phi}}\right] \quad (54)$$

Equations (52) and (53) imply in turn that the following relationships hold, which describe the structural relation between the two systems (i.e., the system in task space error and the original dynamic system in ϕ) given in Eqs. (23) and (29), respectively.

$$[B]\Psi=\partial(\phi,\dot{\phi})\tilde{G} \quad (55)$$

$$[A]T+[B]\Upsilon=\partial(\phi,\dot{\phi})\tilde{F}$$

$$+\left[\frac{\partial T}{\partial u_d}\frac{\partial T}{\partial\dot{u}_d}\right]\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix} \quad (56)$$

3.2 Design of controller

The required control action T_ϕ will be found in the form

$$T_\phi=T^s_\phi+T^r_\phi \quad (57)$$

where T^s_ϕ, $T^r_\phi\in R^n$ denotes stabilizing control and robust control action, respectively. T^s_ϕ is defined first as

$$T^s_\phi=\Psi^{-1}[[K_v]\dot{\varepsilon}+[K_p]\varepsilon-\Upsilon] \quad (58)$$

where $[K_v]$, $[K_p]\in R^{m\times m}$ are derivative and proportional gain matrices used for the same reasons as in the resolved acceleration scheme of the previous section. If there are no modeling errors, i.e., ΔF and ΔG are null matrices, then proper selection of feedback gain matrices $[K_v]$,$[K_p]$ will guarantee asymptotically stable error response. If this is the case, Eqs. (29), (47), and (58) imply with

$$\frac{d}{dt}\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix}=[\hat{A}]\begin{pmatrix}\varepsilon\\ \dot{\varepsilon}\end{pmatrix} \quad (59)$$

where

$$[\hat{A}]=\begin{bmatrix}[0] & [I]\\ [K_p] & [K_v]\end{bmatrix} \quad (60)$$

Letting $[K_v]=diag(k_v), k_v>0$, $[K_p]=diag(k_p)$, $k_p>0$, for example, asymptotic stability can be obtained.

Now we proceed to find the additional control effort T^r_ϕ which will render the system robust under disturbances due to modeling errors. Here, the main issue is to find proper bounding functions which will define the upper bound of the system uncertainties due to the modeling errors.

First, two functions $\Delta\mathcal{F}(\phi,\dot{\phi})\in R^{2m\times 2n}$, $\Delta\mathcal{G}(\phi)\in R^{2m\times n}$ are defined. But for the system under consideration there exist such functions as will be shown shortly.

$$\partial(\phi,\dot{\phi})\Delta F=[B]\Delta\mathcal{F} \quad (61)$$

$$\partial(\phi,\dot{\phi})\Delta G=[B]\Delta\mathcal{G}\,\Psi \quad (62)$$

Noting that we have from Eqs. (43) and (54)

$$\partial(\phi,\dot{\phi})=\left[\frac{\partial T}{\partial\phi}\frac{\partial T}{\partial\dot{\phi}}\right] \quad (63)$$

$$=\begin{bmatrix}[\tilde{G}^u_\phi] & [0]\\ \dot{\phi}^T\oplus\{\tilde{H}^u_{\phi\phi}\} & [\tilde{G}^u_\phi]\end{bmatrix} \quad (64)$$

$\Delta\mathcal{F}$ and $\Delta\mathcal{G}$ can be found by using Eqs. (27), (28), (48) as follows:

$$\Delta\mathcal{F}=[\tilde{G}^u_\phi][I^*_{\phi\phi}]^{-1}\Big([\Delta I^*_{\phi\phi}(\phi)][\tilde{I}^*_{\phi\phi}(\phi)]^{-1}\tilde{N}(\phi,\dot{\phi}) - \Delta N(\phi,\dot{\phi})\Big) \quad (65)$$

$$\Delta\mathcal{G}=-[\tilde{G}^u_\phi][I^*_{\phi\phi}]^{-1}[\Delta I^*_{\phi\phi}(\phi)][\tilde{G}^u_\phi]^{-1} \quad (66)$$

where $[\Delta I^*_{\phi\phi}(\phi)]\in R^{n\times n}$, $\Delta N(\phi,\dot{\phi})\in R^n$ are defined as

$$[\Delta I^*_{\phi\phi}(\phi)]=[I^*_{\phi\phi}(\phi)]-[\tilde{I}^*_{\phi\phi}(\phi)] \quad (67)$$

$$\Delta N(\phi,\dot{\phi})=N(\phi,\dot{\phi})]-\tilde{N}(\phi,\dot{\phi}) \quad (68)$$

Define $[\Gamma(\phi)]\in R^{n\times n}$ and $\Omega(\phi,\dot{\phi},\ddot{u}_d)\in R^n$ as

$$[\Gamma(\phi)]=\Bigg[[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi] + \Big[[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big]^T\Bigg]/2 \quad (69)$$

$$=-\Bigg[[I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)] + \Big([I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)]\Big)^T\Bigg]/2 \quad (70)$$

$$\Omega(\phi,\dot{\phi},\dot{u}_d,\ddot{u}_d)=[\tilde{G}^u_\phi]^{-1}(\Delta\mathcal{G}([K_v]\dot{\varepsilon} + [K_p]\varepsilon-\Upsilon)+\Delta\mathcal{F}) \quad (71)$$

$$=[\tilde{G}^u_\phi]^{-1}\Big\{[\tilde{G}^u_\phi][I^*_{\phi\phi}(\phi)]^{-1}\cdot \Big([\Delta I^*_{\phi\phi}(\phi)][\tilde{I}^*_{\phi\phi}(\phi)]\tilde{N}(\phi,\dot{\phi})-\Delta N(\phi,\dot{\phi})\Big) -[\tilde{G}^u_\phi][I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)][\tilde{G}^u_\phi]^{-1}([K_v]\dot{\varepsilon} +[K_p]\varepsilon-\Upsilon)\Big\} \quad (72)$$

$$=-[I^*_{\phi\phi}(\phi)]^{-1}\Big([\Delta I^*_{\phi\phi}(\phi)]\Sigma +\Delta N(\phi,\dot{\phi})\Big) \quad (73)$$

Where $\Sigma(\phi,\dot{\phi},u,\dot{u},\ddot{u})\in R^n$ is defined

$$\Sigma=[\tilde{G}^u_\phi]^{-1}\Big([K_v]\dot{\varepsilon}+[K_p]\varepsilon-\dot{\phi}^T\otimes\{\tilde{H}^u_{\phi\phi}\} \otimes\dot{\phi}+\binom{\ddot{u}_d}{\ddot{u}_d}\Big) \quad (74)$$

Notice that Σ is usual joint space commanded acceleration, which contains basic proportional and derivative control action about the errors in task space.

Let $\sigma_m(\cdot)$ denote the minimum of the real part of the eigenvalues of a matrix. Now, we assume the following.

Assumption 1 *There exists a C_o function $\alpha(\phi)\in R^+$ such that*

$$\sigma_m([I]+[\Gamma(\phi)])\geq\alpha(\phi) \quad (75)$$

and $\alpha(\phi)$ is bounded by $\alpha_o\in R^+$, i.e., $\alpha_o>\alpha(\phi)>0$ for all $\phi\in R^n$.

Assumption 2 *There exists a C_o function $\beta(\phi,\dot{\phi},u_d,\dot{u}_d,\ddot{u}_d)\in R^+$ such that*

$$\alpha(\phi)\beta(\phi,\dot{\phi},u_d,\dot{u}_d,\ddot{u}_d)\geq\|\Omega(\phi,\dot{\phi},\dot{u}_d,\ddot{u}_d)\|_2 \quad (76)$$

We show that the assumption 2 can be satisfied easily by introducing two additional bounding functions.

Proposition 5 *Let two functions $\gamma(\phi)$, $\delta(\phi,\dot{\phi})\in R^+$ be defined as*

$$\|[I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)]\|_2\leq\gamma(\phi) \quad (77)$$

$$\|[I^*_{\phi\phi}(\phi)]^{-1}\Delta N(\phi,\dot{\phi})\|_2\leq\delta(\phi,\dot{\phi}) \quad (78)$$

Then, $\beta(\phi,\dot{\phi},u_d,\dot{u}_d,\ddot{u}_d)$ defined as

$$\beta=(\gamma\|\Sigma\|_2+\delta)/\alpha \quad (79)$$

satisfies assumption 2.

The proof of this assertion is immediate.

$$\|\Omega\|_2=\|[I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)]\Sigma +[I^*_{\phi\phi}(\phi)]^{-1}\Delta N(\phi,\dot{\phi})\|_2 \quad (80)$$

$$\leq\|[I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)]\|_2\|\Sigma\|_2 +\|[I^*_{\phi\phi}(\phi)]^{-1}\Delta N(\phi,\dot{\phi})\|_2 \quad (81)$$

$$\leq\gamma\|\Sigma\|_2+\delta \quad (82)$$

$$=\alpha\beta \quad (83)$$

Now the rigorous definition on the type of stability concerned in this paper is given below (La Salle and Lefschetz, 1961; Leitman, 1981).

Definition 1 *The tracking error is said to be* uniformly ultimately bounded *$w.r.t.$ u_d, with bound ν iff for every ϵ, $t_o\in R^+$, there exists $\tau(\epsilon,t_o)<\infty$ such that $\|\varepsilon(t_o)\|_2\leq\epsilon$ implies $\|\varepsilon(t)\|_2\leq\nu$ for $t\geq t_o+\tau$. If τ can be selected as a function of ϵ only, the bound is said to be uniform $w.r.t.$ t and u_d.*

Related to this definition, we have the following main theorem.

Theorem 1 *Suppose that the assumptions 1 and 2 are satisfied, that gain matrices $[K_v]$,$[K_p]$ are chosen such that the real parts of the eigenvalues of the matrix defined by*

$$[\hat{A}]=\begin{bmatrix}[0] & [I]\\ [K_p] & [K_v]\end{bmatrix} \quad (84)$$

are all negative (i.e., $[\hat{A}]$ is Herwitz), and finally that the positive definite matrix $[P] \in R^{2m\times 2m}$ is a unique solution of following Lyapunov equation

$$[P][\hat{A}]+[\hat{A}]^T[P]=-[Q] \tag{85}$$

where $[Q] \in R^{2m\times 2m}$ is any properly selected positive definite matrix. Then, the control action $T_\phi = T_\phi^s + T_\phi^r$ will drive the system given in Eq. (1) such that $\|\varepsilon(t)\|_2$ (see Eq. (42)) is to be uniformlyultimately bounded *for $t \in [t_o, \infty)$, where T_ϕ^s, T_ϕ^r are found by*

$$T_\phi^s = \Psi^{-1}[[K_v]\dot{\varepsilon}+[K_p]\varepsilon - \Upsilon] \tag{86}$$

$$=\tilde{N}(\phi,\dot{\phi})+[\tilde{I}^*_{\phi\phi}(\phi)]\Sigma \tag{87}$$

and

$$T_\phi^r = -\beta\Psi^{-1}[\tilde{G}_\phi^u]sat(z) \tag{88}$$

$$=-\beta[\tilde{I}^*_{\phi\phi}(\phi)]sat(z) \tag{89}$$

with saturation function $sat(\cdot)$ being defined as

$$sat(z)=\begin{cases} z & if \|z\|_2 \le 1 \\ z/\|z\|_2 & if \|z\|_2 > 1 \end{cases} \tag{90}$$

and where the control vector $z(\varepsilon) \in R^{2m}$ is defined by

$$z=\alpha_o\beta[\tilde{G}_\phi^u]^T[B]^T[P]T(\phi,\ \dot{\phi},\ u_d,\ \dot{u}_d) \tag{91}$$

The proof for the theorem is somewhat lengthy but straightforward as shown in Appendix (see also Gilbert and Ha (1984) for the similar reasoning). The primary task is to find the error dynamics in the form which contains two distinct parts: one showing asymptotically stable behavior and the other, describing the disturbance due to the modeling discrepancies, expressed in terms of previously defined quantities, e.g., ΔF, ΔG, etc.. Once this is done, a Lyapunov-like function is introduced to show the boundedness of the tracking error.

We also have the following corollary which gives an explicit envelop function in time of the error $\|\varepsilon(t)\|_2$.

Corollary 1 Assume all the assumptions made in Theorem 1 remain valid. Then, the tracking error $\|\varepsilon(t)\|_2$ is bounded for $t \in [t_o, \infty)$ as

$$\|\varepsilon(t)\|_2 \le \begin{cases} \xi\epsilon & if\ \epsilon_o \le 1 \\ \xi\epsilon\{1+(\epsilon_o^2-1)e^{-\delta_m(t-t_o)}\}^{1/2} & if\ \epsilon_o > 1 \end{cases} \tag{92}$$

where $\xi=(\sigma_m([P]))^{-1/2}$, $\epsilon = 1/(2\delta_m^{1/2})$, and $\epsilon_o = 2\delta_m^{1/2}\|[P]^{1/2}\ x(t_0)\|_2$ with x being defined as $x^T = (\varepsilon^T,\ \dot{\varepsilon}^T)$.

Multiplying both sides of Eq. (161) by the integrating factor $e^{\delta_m t}>0$, we have

$$(\dot{V}+\delta_m V)e^{\delta_m t} \le e^{\delta_m t}/4 \tag{93}$$

Integrating both sides of Eq. (93) gives

$$V \le \frac{1}{4\delta_m}+\left(V(t_o)-\frac{1}{4\delta_m}\right)e^{-\delta_m(t-t_o)} \tag{94}$$

Noting that $\varepsilon=[C]x$ where $[C]=[[I][O]]$, the assertion easily follows from the fact that $\|\varepsilon\|_2 \le \|x\|_2 \le \|[P]^{-1/2}\|_2\|[P]^{1/2}x\|_2 = \|[P]^{-1/2}\|_2 V^{1/2}$.

Some discussions on the implications of Corollary 1 will be made in what follows (see also Gilbert and Ha, 1984). Corollary 1 shows that the asymptotic bound on the tracking error $\|\varepsilon\|_2$ is given by ξ and ϵ which are strongly dependent on $[P]$, the gain matrix used in evaluating control vector z (see Eq. (91)). It can be observed from the definition of ξ that to reduce the magnitude of ξ requires a large gain matrix $[P]$ in the sense of norm. Noting that $[P]$ is found by solving the Lyapunov equation, it is required to choose a large matrix $[Q]$ for a given $[\hat{A}]$ to increase the magnitude of matrix $[P]$. However, there is no direct way to reduce the magnitude of ϵ. This can be seen from the definition $\delta_m = \sigma_m([P]^{-1}[Q])$. This shows that the increase in magnitude of $[P]$ resulting from large $[Q]$ will be cancelled thereby causing little change in ϵ. One way to increase δ_m is to use a large matrix $[\hat{A}]$. In fact, the Lyapunov equation implies

$$\|[Q]\|_2/\|[P]\|_2 \le 2\|[\hat{A}]\|_2 \tag{95}$$

So large $\|[\hat{A}]\|_2$, which essentially requires large stabilizing feedback gain matrices $[K_v]$ and $[K_p]$, would help to increase δ_m thereby reducing ϵ.

3.3 Practical implementation of the proposed controller

From a practical point of view, the proposed controller described in Theorem 1 cannot be easily implemented especially when the order of the system is large, as in general multi-degree of freedom robotic manipulators. This fact can be appreciated by recalling the Assumptions 1, 2, and related Proposition 5. They require off-line evaluation of the positive constant α_o and construction of the positive functionals $\alpha(\phi)$, $\gamma(\phi)$, and $\delta(\phi, \dot{\phi})$. Although the existence of those functionals is rather obvious, the actual off-line construction of them may be difficult for the general class of nonlinear systems. This makes it inevitable to estimate those required bounding functions on-line. This issue will be addressed in the following.

First, consider on-line construction of the functionals $\gamma(\phi)$, and $\delta(\phi, \dot{\phi})$. The on-line estimation of $\delta(\phi, \dot{\phi})$ is simple. Noting that the 2-norm of a vector is easy to evaluate, we may simply set

$$\delta(\phi, \dot{\phi}) = \|[I^*_{\phi\phi}(\phi)]^{-1}\Delta N(\phi, \dot{\phi})\|_2 \quad (96)$$

The estimation of $\gamma(\phi)$ in similar fashion, however, is not feasible because the evaluation of the 2-norm of a matrix, which essentially amounts to finding the maximum singular value of the matrix, is computationally demanding especially for on-line processing. For the on-line estimation of $\gamma(\phi)$, we may use any one of the following relations, which essentially yields an over-estimate for the desired 2-norm of a matrix $[A]\in R^{n\times n}$ at the expense of computational ease.

$$\|[A]\|_2 \le \sqrt{n}\|[A]\|_\infty \quad (97)$$
$$\|[A]\|_2 \le \sqrt{n}\|[A]\|_1 \quad (98)$$
$$\|[A]\|_2 \le \|[A]\|_F \quad (99)$$

where

$$\|[A]\|_1 = \max_j \sum_{i=1}^{n}|A_{ij}| \quad (100)$$

$$\|[A]\|_\infty = \max_i \sum_{i=1}^{n}|A_{ij}| \quad (101)$$

$$\|[A]\|_F = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}|A_{ij}|^2\right)^{1/2} \quad (102)$$

Noting from Eqs. (79) and (58) that the more tight the estimation of $\gamma(\phi)$ is, the less control torque will result, then the following proposition which is too simple to prove may be used to single out a proper norm for this purpose.

Proposition 6 *For a given matrix $[A]\in R^{n\times n}$, following inequality relations hold:*

$$\|[A]\|_2 \le \|[A]\|_F \le \sqrt{n}\|[A]\|_\infty \quad (103)$$

$$\|[A]\|_2 \le \|[A]\|_F \le \sqrt{n}\|[A]\|_1 \quad (104)$$

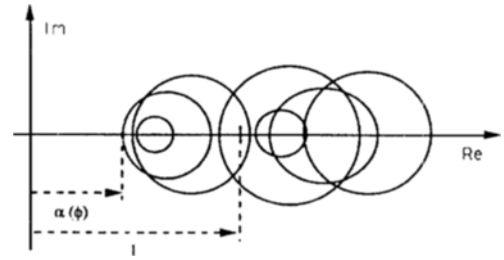


Fig. 2 Greatest lower bound of gerschgorin disks

In view of Proposition 6, $\gamma(\phi)$ is estimated on-line as

$$\gamma(\phi) = \|[I^*_{\phi\phi}(\phi)]^{-1}[\Delta I^*_{\phi\phi}(\phi)]\|_F \quad (105)$$

In connection with the on-line estimate of $\alpha(\phi)$ we have the following proposition, which is a direct implication of well known Gerschgorin circle theorem (Lancaster, 1969).

Proposition 7 *Assume $[[I]+[\Gamma(\phi)]]$ is strictly diagonally dominant in the sense that*

$$|1+\Gamma_{ii}| > \sum_{\substack{j=1\\ j\ne i}}^{n}|\Gamma_{ij}| \qquad i=1, \ldots, n \quad (106)$$

Then, $\alpha(\phi)$ given by

$$\alpha(\phi) = \inf\{y : y \in \cup_i D^g_i\} \quad (107)$$

where the Gerschgorin discs $D^g_i (i=1, \ldots, n)$ is defined as

$$D^g_i = \left\{ y : |1+\Gamma_{ii} - y| \le \sum_{\substack{j=1\\ j\ne i}}^{n}|\Gamma_{ij}| \right\} \quad i=1, \ldots, n \quad (108)$$

satisfies

$$\sigma_m([I]+[\Gamma(\phi)]) \ge \alpha(\phi) \quad (109)$$

For the clear explanation of the implication made by the proposition see Fig. 2.

Finally, by letting $\alpha_o = \alpha(\phi)$, where $\alpha(\phi)$ is defined in Eq. (107), it is easy to see that the same transition from Eq. (160) to Eq. (161) can be made thereby leading to the same conclusion given in Theorem 1.

3.4 Simulation and results

In this section some numerical simulation results are provided to demonstrate the effectiveness of the proposed controller for the control of a MAMI system. For the simulation, the parallel type hybrid robotic manipulator system is used. In the simulation we assumed the following system parameters: the length of a large link was assumed to be $1m$ and that of a small link $5cm$. The mass of a large link was taken as $20kg$ with the center of mass being located at its geometric center. In proportion to the link ratio $5kg$ was used as the mass of a small link. Inertia matrix of a link was evaluated by assuming the link is a thin uniform bar. Each side length of upper ternary was assumed to be $1m$. The mass of upper ternary was assumed to be $30kg$ with its geometric center being the center of the mass. The inertia matrix of the upper ternary was calculated by assuming it is a uniform triangle. In the simulation we assumed that due to the variation of payload there is 20% uncertainty of the mass of upper ternary (i.e. endeffector).

The stabilizing feedback gain matrices $[K_p] \in R^{6\times6}$ and $[K_v] \in R^{6\times6}$ were chosen as follows: $[K_p]_{ij} = [K_v]_{ij} = 0$ if $i \neq j$ and

$$[K_p]_{ii} = diag(-\lambda_i^2) \qquad (i=1, ..., 6) \quad (110)$$

$$[K_v]_{ii} = diag(2\lambda_i) \qquad (i=1, ..., 6) \quad (111)$$

where $\lambda_i \in R^+$. Notice that this particular selection of the stabilizing feedback gain matrices leads the matrix $[\hat{A}]$ to have double (real) eigenvalues of $-\lambda_i (i=1, ..., 6)$. This implies that if there is no modeling discrepancies, a typical error behavior under unit impulse can be expressed in the form

$$\varepsilon_i(t) = (c_{1i} + c_{2i}t)e^{-\lambda_i t} \qquad (i=1, ..., 6) \quad (112)$$

where c_{1i} and c_{2i} are constants determined by the initial conditions.

Next, to find the Lyapunov gain matrix $[P]$ we define $[Q]$ as

$$[Q] = \begin{bmatrix} [diag\ \lambda_i] & [O] \\ [O] & [I] \end{bmatrix} \quad (113)$$

where each submatrix are of $R^{6\times6}$. This together with Eqs. (110) and (111) allows one to find the Lyapunov gain matrix $[P]$ from Eq. (85) in closed form as

$$[P] = \frac{1}{2}\begin{bmatrix} [diag\ -3\lambda_i] & [I] \\ [I] & [diag\ -\lambda_i^{-1}] \end{bmatrix} \quad (114)$$

In the simulations following values of $\lambda_i, i=1, ..., 6$ were used.

$$\lambda_i = 10, \qquad i=1, ..., 3 \quad (115)$$

$$\lambda_i = 5, \qquad i=4, ..., 6 \quad (116)$$

Figures 3 through 12 show some results of the simulation where the desired end-effector motion is planned as

$$x_d(t) = 0.15(1-\cos(2\pi ft)) \quad (0 \leq t \leq 2) \quad (117)$$

$$y_d(t) = 0.15(1-\cos(2\pi ft)) \quad (0 \leq t \leq 2) \quad (118)$$

$$\theta_d(t) = 0 \quad (119)$$

where the motion frequency f was chosen to be 0.5 Hz. This implies that the desired average speed of end-effector is about $0.3m/sec$ in both forward and returning motions.

Various performances of the proposed controller are compared with those obtained from conventional resolved acceleration technique (Luh et al., 1980). Figures 3 to 8 show that robust controller outperforms the non-robust controller. This can be clearly seen during the interval $1\ sec \leq t \leq 2sec$. Since the residual momentum due to the modeling discrepancies at about $t=1sec$ severely perturbs the control action of conventional resolved acceleration scheme for the returning motion, much larger errors than those of robust controller result.

Figures 3 to 5 depict the x, y, and orientation-error at end-effector. Figures 6, 7, and 8 show that proposed robust controller restricts the excursion of jointsassociated with small links to smaller ranges (as it is desired to prevent the joint saturations associated with small links) than those

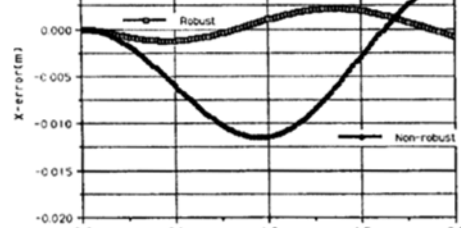


Fig. 3 X-direction error at end-effector

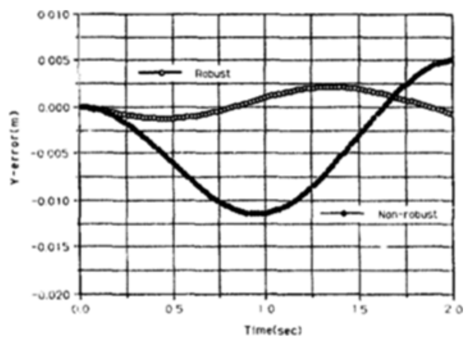


Fig. 4 Y-direction error at end-effector

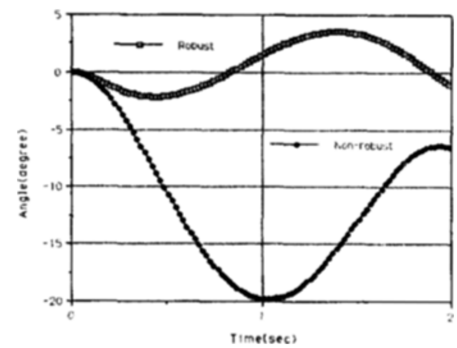


Fig. 6 Motion of small joint ϕ_2

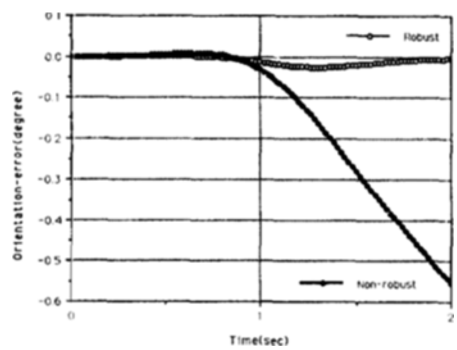


Fig. 5 Orientation error at end-effector

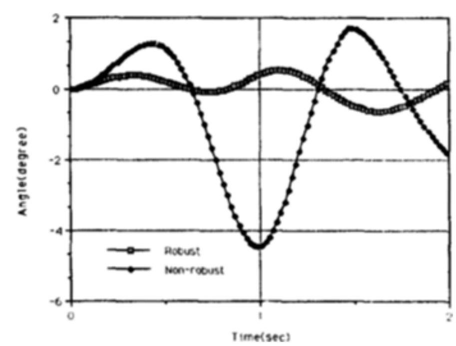


Fig. 7 Motion of small joint ϕ_4

produced by resolved acceleration controller.

Required torque history for the joints associated with small links during the motion are given in Figs. 9 to 11. Small increase of control efforts in robust controller than non-robust one can be observed especially when modeling errors prevail (i.e. around $t=1sec$) as shown in Fig. 12. In fact, proposed controller generally but not necessarily requires more control efforts than non-robust controller. This statement may be substantiated by recalling the expressions of T^s_ϕ and T^z_ϕ (see Eq. (87) and (89)).

4. Conclusion

This paper proposes a robotic manipulator system called hybrid type macro-micro system which contains two sub-robotic systems so that a large but coarse motion is generated by macro-robot while a small and accurate motion is created by micro-manipulator. The overall robotic manipulator system with this structure possesses distinct advantage over the conventional one in that high precision motion at end-effector can be achievd while maintaining large overall work-space.

However, it is important to realize that the controller for macor-micro robotic system should be very robust. This is because the dynamic interations between two sub-systems must be suppressed properly so that each sub-system can perform its own task without serious dynamic influence from the other. For this purpose, this paper also proposes a robust controller for high precision robotic manipulators of hybrid type. It should also be noted that some concerns to prevent frequent joint saturation at small links (i.e., at micor-robotic system) should be made because micro-robot system, which has larger dynamic bandwidth than macro-robotsystem, may be easily saturated for large positon and orientation errors at end-effector. The controller proposed in this paper takes this point into account by

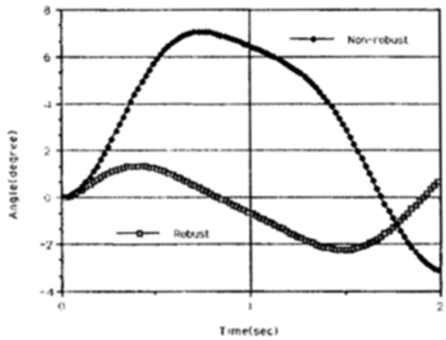


Fig. 8 Motion of small joint ϕ_6

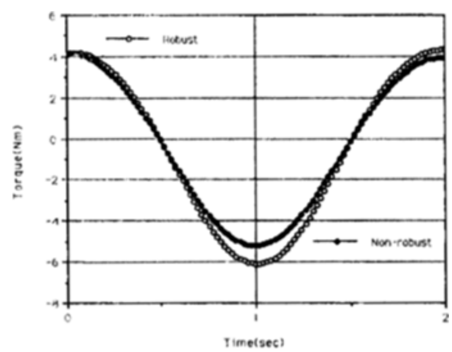


Fig. 9 Required torque at ϕ_2

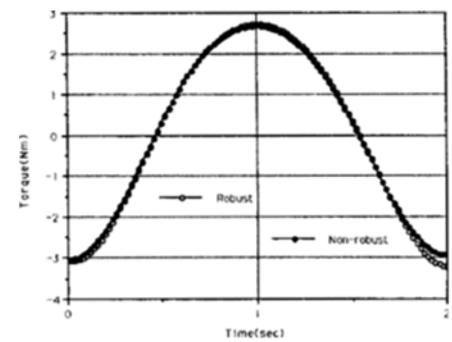


Fig. 10 Required torque at ϕ_4

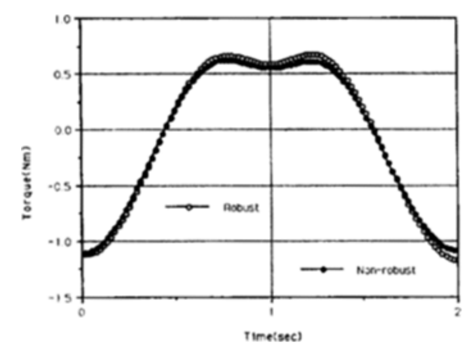


Fig. 11 Required torque at ϕ_6

predefining nominal home configuration for the micro-robot and preventing large excursions from home configuration.

Appendix

To prove Theorem 1, let's define $x^T=(\varepsilon^T, \dot{\varepsilon}^T) = T(\phi, \dot{\phi}, u_d, \dot{u}_d)$. Then, we have by differentiation w.r.t. time

$$\dot{x} = \partial(\phi, \dot{\phi})\begin{pmatrix}\dot{\phi}\\ \ddot{\phi}\end{pmatrix} + \left[\frac{\partial T}{\partial u_d} \frac{\partial T}{\partial \dot{u}_d}\right]\begin{pmatrix}\dot{u}_d\\ \ddot{u}_d\end{pmatrix} \ (\Leftarrow(51)) \qquad (120)$$

$$= \partial(\phi, \dot{\phi})[F+GT_\phi] \ (\Leftarrow(18)) + [A]x+[B]\Upsilon - \partial(\phi, \dot{\phi})\tilde{F} \ (\Leftarrow(56)) \qquad (121)$$

$$= \partial(\phi, \dot{\phi})\Delta F + [A]x+[B]\Upsilon + \partial(\phi, \dot{\phi})G(T_\phi^s + T_\phi^r) \qquad (122)$$

Noting that

$$[B]\Upsilon = [B]\Psi\Psi^{-1}\Upsilon \qquad (123)$$

$$= \partial(\phi, \dot{\phi})\tilde{G}\Psi^{-1}\Upsilon \ (\Leftarrow(55)) \qquad (124)$$

and

$$T_\phi^s = \Psi^{-1}[[K]x - \Upsilon] \ (\Leftarrow(58)) \qquad (125)$$

where $[K]\in R^{m\times 2m}$ is defined as

$$[K]=[[K_p][K_v]] \qquad (126)$$

we have

$$\dot{x} = [A]x + \partial(\phi, \dot{\phi})\Delta F - \partial(\phi, \dot{\phi})\Delta G\Psi^{-1}\Upsilon + \partial(\phi, \dot{\phi})G\Psi^{-1}[K]x + \partial(\phi, \dot{\phi})GT_\phi^r \qquad (127)$$

Adding and subtracting $[B][K]x$ from R.H.S. of Eq. (127), we manipulate further as follows:

$$\dot{x} = [\hat{A}]x + \partial(\phi, \dot{\phi})\Delta F - \partial(\phi, \dot{\phi})\Delta G\Psi^{-1}\Upsilon + (\partial(\phi, \dot{\phi})G\Psi^{-1} - [B])[K]x + \partial(\phi, \dot{\phi})GT_\phi^r \qquad (128)$$

$$= [\hat{A}]x + [B]\Delta\mathcal{F} - [B]\Delta\mathcal{G}\Upsilon \ (\Leftarrow(61), (62)) + \left[\partial(\phi, \dot{\phi})G\Psi^{-1} - [B]\right][K]x + \partial(\phi, \dot{\phi})GT_\phi^r \qquad (129)$$

$$= [\hat{A}]x + [B]\Delta\mathcal{F} - [B]\Delta\mathcal{G}\Upsilon + \partial(\phi, \dot{\phi})GT_\phi^r$$

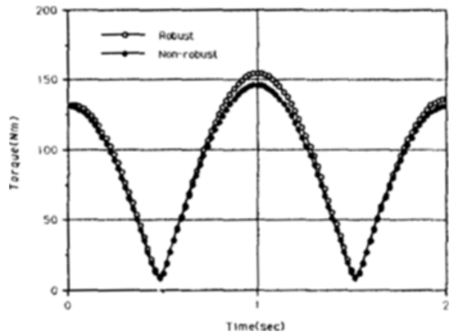


Fig. 12 Norm of the total torque $\|T\phi\|_2$

$$+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-\partial(\phi,\dot\phi)\tilde G\Psi^{-1}+\partial(\phi,\dot\phi)G\Psi^{-1}-[B]\Big][K]x \tag{130}$$

$$=[\hat A]x+[B]\Delta\mathcal{F}-[B]\Delta\mathcal{G}\gamma+\partial(\phi,\dot\phi)GT_\phi^r+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-[B]\Big][K]x+\partial(\phi,\dot\phi)\Delta G\Psi^{-1}[K]x \tag{131}$$

$$=[\hat A]x+[B]\Delta\mathcal{F}-[B]\Delta\mathcal{G}\gamma+[B]\Delta\mathcal{G}[K]x\ (\Leftarrow(62))+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-[B]\Big][K]x+\partial(\phi,\dot\phi)GT_\phi^r \tag{132}$$

$$=[\hat A]x+[B](\Delta\mathcal{F}+\Delta\mathcal{G}([K]x-\gamma))+\partial(\phi,\dot\phi)GT_\phi^r+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-[B]\Big][K]x \tag{133}$$

$$=[\hat A]x+[B]\tilde G\Omega\ (\Leftarrow(71))+\partial(\phi,\dot\phi)G\Psi^{-1}\Psi T_\phi^r+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-[B]\Big][K]x \tag{134}$$

$$=[\hat A]x+[B]\tilde G\Omega+\Big[\partial(\phi,\dot\phi)\tilde G\Psi^{-1}-[B]\Big][K]x+\partial(\phi,\dot\phi)\Big[G\Psi^{-1}\Psi-\tilde G\Psi^{-1}\Psi+\tilde G\Psi^{-1}\Psi\Big]T_\phi^r \tag{135}$$

$$=[\hat A]x+[B]\tilde G\Omega+[B]\Delta\mathcal{G}\,\Psi T_\phi^r+\partial(\phi,\dot\phi)\tilde G\,\Psi^{-1}(\Psi T_\phi^r+[K]x)-[B][K]x\ (\Leftarrow(62)) \tag{136}$$

$$=[\hat A]x+[B]\tilde G\Omega++[B]\Delta\mathcal{G}\,\Psi T_\phi^r+[B](\Psi T_\phi^r+[K]x)-[B][K]x\ (\Leftarrow(55)) \tag{137}$$

$$=[\hat A]x+[B]\tilde G\Omega+[B]\Delta\mathcal{G}\,\Psi T_\phi^r+[B]\Psi T_\phi^r \tag{138}$$

$$=[\hat A]x+[B]\tilde G\Omega+[B][[I]+\Delta\mathcal{G}]\Psi T_\phi^r \tag{139}$$

By premultiplying $[\tilde G_\phi^u]$ and postmultiplying $[\tilde G_\phi^u]^{-1}$ to the last term, we finally arrive at desired fined form as

$$\dot x=[\hat A]x+[B]\tilde G\Omega+[B][\tilde G_\phi^u]\Big[[I]+[\tilde G_\phi^u]^{-1}\Delta\mathcal{G}[\tilde G_\phi^u]\Big][\tilde G_\phi^u]^{-1}\Psi T_\phi^r \tag{140}$$

To see the behavior of the system given in Eq. (140) we define the following positive definite function $V(x)$ as

$$V=x^T[P]x/2 \tag{141}$$

From this and Eq. (140) we find the time rate of change of $V(x)$ as

$$\dot V=x^T[P]\dot x \tag{142}$$

$$=x^T[P][\hat A]x+x^T[P][B][\tilde G_\phi^u]\Omega+x^T[P][B][\tilde G_\phi^u]\Big[[I]+[\tilde G_\phi^u]^{-1}\Delta\mathcal{G}[\tilde G_\phi^u]\Big][\tilde G_\phi^u]^{-1}\Psi T_\phi^r \tag{143}$$

We will consider the three terms of R.H.S. of Eq. (143) separately. Consider the first term: Since $x^T[P][\hat A]x=x^T[\hat A]^T[P]x$, Eq. (85) implies

$$x^T[P][\hat A]x=-x^T[Q]x/2 \tag{144}$$

Recalling the following well known fact for any two positive matrices $[P]$ and $[Q]$

$$0<\delta_m\le\frac{x^T[Q]x}{x^T[P]x}\le\delta_M \tag{145}$$

where $\delta_m=\sigma_m([P]^{-1}[Q])$ and $\delta_M=\sigma_M([P]^{-1}[Q])$, we may conclude

$$x^T[P][\hat A]x=-x^T[Q]x/2 \tag{146}$$

$$\le-\frac{\delta_m}{2}x^T[P]x \tag{147}$$

$$=-\delta_m V \tag{148}$$

Next, consider the second term of Eq. (143): By using Eq. (91) and Schwartz inequality, we have

$$|x^T[P][B][\tilde G_\phi^u]\Omega|=|z\Omega/\alpha_o\beta|\ (\Leftarrow(91)) \tag{149}$$

$$\le\|z\|_2\|\Omega\|_2/\alpha_o\beta \tag{150}$$

where $|\cdot|$ denotes the absolute value of a number. This in turn implies via Assumptions 1 and 2

$$|x^T[P][B][\tilde G_\phi^u]\Omega|\le\alpha\|z\|_2/\alpha_o \tag{151}$$

Consider finally the last term of Eq. (143). Using Eqs. (88) through (91) we can find

$$x^T[P][B][\tilde{G}^u_\phi]\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big]\cdot[\tilde{G}^u_\phi]^{-1}\Psi T^T_\phi= \tag{152}$$

$$\begin{cases} -\alpha_o(\beta)^2x^T[P][B][\tilde{G}^u_\phi]\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big][\tilde{G}^u_\phi]^T[B]^T[P]x & \text{if } \|z\|_2\le 1 \\ -\alpha_o(\beta)^2x^T[P][B][\tilde{G}^u_\phi]\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big][\tilde{G}^u_\phi]^T[B]^T[P]x/\|z\|_2 & \text{if } \|z\|_2>1 \end{cases} \tag{153}$$

$$=-\rho(z)\alpha_o(\beta)^2x^T[P][B][\tilde{G}^u_\phi]\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big]\cdot [\tilde{G}^u_\phi]^T[B]^T[P]x \tag{154}$$

where $\rho(z)$ is defined for compactness purpose as

$$\rho(z)=\begin{cases}1 & \text{if } \|z\|_2\le 1\\ \|z\|_2^{-1} & \text{if } \|z\|_2>1\end{cases} \tag{155}$$

Using Eq. (91), we can rewrite Eq. (154) as

$$x^T[P][B][\tilde{G}^u_\phi]\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big][\tilde{G}^u_\phi]^{-1}\Psi T^T_\phi$$
$$=-\rho(z)\alpha_o^{-1}z^T\Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big]z \tag{156}$$

In view of Eqs. (148), (151), and (156), Eq. (143) can be bounded as

$$\dot{V}\le-\delta_m V+\alpha\alpha_o^{-1}\|z\|_2-\rho(z)\alpha_o^{-1}z^T \Big[[I]+[\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big]z \tag{157}$$
$$\le-\delta_m V+\alpha\alpha_o^{-1}\|z\|_2-\rho(z)\alpha_o^{-1}\cdot \Big(1+\sigma_m\Big([\tilde{G}^u_\phi]^{-1}\Delta\mathcal{G}[\tilde{G}^u_\phi]\Big)\Big)\|z\|_2^2 \tag{158}$$
$$\le-\delta_m V+\alpha\alpha_o^{-1}\|z\|_2-\rho(z)\alpha\alpha_o^{-1}\|z\|_2^2 \tag{159}$$
$$=-\delta_m V+\alpha\alpha_o^{-1}(\|z\|_2-\rho(z)\|z\|_2^2) \tag{160}$$

During the above successive bounding process the use of the Rayleigh quotient and Assumption 1 were made. Now, noting that the terms in the parenthesis in Eq. (160) has maximum value of 1/4, we can finally conclude

$$\dot{V}\le-\delta_m V+\frac{1}{4} \tag{161}$$

We need the following theorem (La Salle and Lefschetz (1961)) to complete the proof.

Theorem 2 *Let $V(x, t)$ be a scalar function with continuous first partial derivatives w.r.t. $x\in R^n$ and t (i.e., $V(x, t)\in C_1$), and let M be a closed subset in R^n with M^c denoting its complement (i.e., $M\cup M^c=R^n$). Then, if $\dot{V}(x, t)\le 0$ for all x and if $V(x_1, t_1)< V(x_2, t_2)$ for all $t_2\ge t_1\ge 0$, all $x_1\in M$ and $x_2\in M^c$, then each solution of $\dot{x}=F(x, t)$, $t\ge 0$ which at some time $t_o\ge 0$ is in M can never thereafter leave M.*

Defining in view of Theorem 2 the set M as

$$M=\{x : \delta_m x^T[P]x\le 1/(4\delta_m)\} \tag{162}$$

we may conclude that the condition given in eq. (161) implies $\|x(t)\|_2$ is uniformly ultimately bounded to the set M. This completes the proof of Theorem 1.

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